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The Diffusion of Ions across Biological Membranes

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Summary. The derivation of a diffusion equation is given for the transport of ions across biological membranes. It is suggested that the diffusion proceeds by the jumps of ions from one site to another and the number of such sites in the membrane is restricted.

In some recent papers treating the ion diffusion across biological membranes a similarity is noticed between this process and diffusion in solids and it is suggested that diffusion across membranes proceeds by the jumps of ions from one binding site to another (Parlin & Eyring, 1954; Agin & Schauf, 1968; Agin, 1969; Volkenstein & Fishman, 1970; Roy, 1971).

If the number of such diffusion centers is large enough as compared with the ion concentration the membrane current has to be expressed as

$$I = -u\,C\frac{d\mu}{dx} \tag{1}$$

where u = ion mobility, C = the concentration of ions in the membrane, x = the coordinate across the membrane, μ = electrochemical potential which is equal to

$$\mu = \mu^0 + RT\ln C + FE. \tag{2}$$

In Eq. (2) E is the electrical potential at point x, F is the Faraday number, T stands for absolute temperature, R is the gas constant, and μ^0 is the standard chemical potential.

When the concentration of ions C is not small as compared with the concentration of sites b, Eq. (2) is not valid. Agin (Agin & Schauf, 1968; Agin, 1969) puts forward the well-founded suggestion that the chemical potential for ideal gas Eq. (2) has to be substituted by the chemical potential for the lattice gas:

$$\mu=\mu^0+RT\ln\frac{C}{b-C}+FE. \tag{3}$$

Insertion of Eq. (3) into Eq. (1) gives the changed Nernst-Planck equation which is used by Agin. His suggestion about the equilibrium ion distribution in a membrane results in the current-voltage curve with a negative slope, but in reality the region with a negative slope lies apparently beyond the region which is near the equilibrium. Here we should like to point out another contradiction – the erroneous insertion of Eq. (3) into Eq. (1). Eq. (3) takes into account the restricted number of diffusion sites, whereas Eq. (1) is valid only if this number is very large.

The recent work by Roy (1971) contains the same contradiction. The author considers the number of diffusion sites to be so small that all of them are occupied by ions, therefore the concentration of ions in the membrane is constant. Then he integrates Eq. (1) with the condition $C(x)=$ const. However, this equation is valid only if the number of sites is large enough.

If we treat the diffusion jumps between a limited number of sites we have to use the following equation:

$$I=-\tilde{u}\,C(b-C)\frac{d\mu}{dx} \tag{4}$$

where $\tilde{u}\,b=u$.

Let us consider one of the simplest proofs of this equation. Let the ion concentration in the neighbor points be C_1 and C_2; the concentration of sites, b_1 and b_2; the rate constants of the reactions of jumps over energetical barrier, k_1 and k_2; and the barrier thickness, λ. Then the flow over this barrier is

$$g=\lambda k_1 C_1(b_2-C_2)-\lambda k_2 C_2(b_1-C_1). \tag{5}$$

Eq. (5) is like the equation derived by Parlin and Eyring (1954). It differs by the factors $(b-C)$ which mean that the number of jumps over the barrier depends on the number of the free sites. If the membrane is homogeneous, $b(x)=$const and $b_1=b_2\equiv b$. When λ is small enough

$$C_2\simeq C_1+\lambda\frac{dC_1}{dx} \tag{6}$$

and

$$k_2=k_1\exp\left[(E_2-E_1)F/RT\right]\simeq k_1\left(1+\frac{\lambda F}{RT}\frac{dE_1}{dx}\right). \tag{7}$$

Insertion of Eqs. (6) and (7) into Eq. (5) gives

$$g = -\lambda^2 k C(b-C)\left[\frac{F}{RT}\frac{dE}{dx} + \frac{b}{C(b-C)}\frac{dC}{dx}\right]. \tag{8}$$

Since $\lambda^2 k b = \mathscr{D}$ ($\mathscr{D}$ = diffusion coefficient; *cf.* Parlin & Eyring, 1954), Eq. (3) may be expressed in the form:

$$g = -\frac{\mathscr{D}}{bRT} C(b-C)\frac{d}{dx}\left[FE + RT\ln\frac{C}{b-C}\right]. \tag{9}$$

The expression in the square brackets of Eq. (9) is the electrochemical potential for the lattice gas [*cf.* Eq. (3)]. The electrical current in the point x is equal to

$$I = Fg = -\frac{F\mathscr{D}}{bRT} C(b-C)\frac{d\mu}{dx} \tag{10}$$

where μ is given by Eq. (3). The ion mobility u is connected with the diffusion coefficient $\mathscr{D}$ by the common formula: $u = F\mathscr{D}/RT$. We can see that Eq. (10) reduces to Eq. (4). This is the simplest equation which treats the diffusion in the medium with the restricted number of diffusion sites. If $b \gg C$, Eq. (10) turns into Eq. (1) and Eq. (3) into Eq. (2).

In the condition of constant field, Eq. (10) may be rewritten in the form

$$I = -\tilde{u}\, b RT\frac{dC}{dx} + u FC(b-C)\frac{V}{a} \tag{11}$$

where V is the potential difference across the membrane and a is the membrane thickness. Eq. (11) may be integrated. The solution gives neither the negative resistance nor a linear $I-V$ curve without additional suggestions about the parameters $\tilde{u}$ and b.

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